M2 June 2011 Written Solutions

① (a) $\frac{1}{2}mu^2 = \frac{1}{2} \times 58 \times 2^2 = 116\text{ J}$

(b) altogether $\Delta h = 6 + 1 = 7\text{m}$

$\Delta GPE = mg\Delta h = 58 \times 9.8 \times 7 = 3978.8\text{ J}$

$E_k = 116 + 3978.8 = 4094.8$

$\frac{1}{2}mv^2 = 4094.8$

$v = \sqrt{\frac{2 \times 4094.8}{58}} = 11.8828$

$v = 11.9\text{ms}^{-1}$

② A $(9,6)$ 2kg　　C $(3,8)$ 8kg

B $(2,4)$ 3kg　　D $(6,11)$ 7kg

Total mass $= 2 + 8 + 3 + 7 = 20\text{kg}$

$20\bar{x} = 9 \times 2 + 2 \times 3 + 3 \times 8 + 6 \times 7$

$20\bar{x} = 90$

$\bar{x} = \frac{9}{2} \quad (= 4.5)$

$20\bar{y} = 6 \times 2 + 4 \times 3 + 8 \times 8 + 11 \times 7 = 165$

$20\bar{y} = 165 \Rightarrow \bar{y} = \frac{33}{4} \quad (= 8.25)$

com is at $(4.5, 8.25)$

③ (a) $a = \frac{dv}{dt} = \left(-2 \times 4e^{-2t}\right)\underline{i} + (6 - 6t)\underline{j}$

$a = -8e^{-2t}\underline{i} + 6(1-t)\underline{j}$

(b) (i) $F = ma = -40e^{-2t}\underline{i} + 30(1-t)\underline{j}$

(ii) $\underline{\underline{t=0}}$ $F = -40e^{0}\underline{i} + 30(1-0)\underline{j}$

$= -40\underline{i} + 30\underline{j}$

magnitude $= \sqrt{(-40)^2 + (30)^2} = 50\,N$

(c) F acts due West $\Rightarrow \underline{\underline{F_{\underline{j}} = 0}}$

so set $30(1-t) = 0$

$\Rightarrow \underline{\underline{t = 1s}}$

(d) $\underline{r} = \int \underline{v}\, dt$

$= \int 4e^{-2t}\, dt\ \underline{i} + \int 6t - 3t^2\, dt\ \underline{j}$

$= \left(\frac{-4}{2}e^{-2t}\right)\underline{i} + \left(3t^2 - t^3\right)\underline{j} + \underline{c}$

$\underline{r} = -2e^{-2t}\underline{i} + (3t^2 - t^3)\underline{j} + \underline{c}$

$\underline{\underline{t=0}}$ $\underline{r} = 6\underline{i} + 5\underline{j}$

$6 = -2e^{-2\times 0} + c_i$

$6 = -2 + c_i$

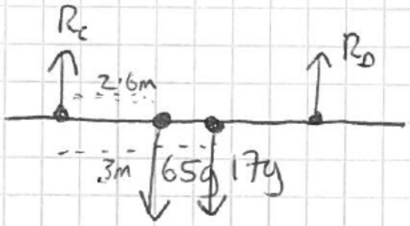
$\Rightarrow \underline{\underline{c_i = 8}}$

$5 = (3(0) - 0) + c_j$

$\Rightarrow \underline{\underline{5 = c_j}}$

$\underline{r} = (-2e^{-2t} + 8)\underline{i} + (3t^2 - t^3 + 5)\underline{j}$

(4) (a)



(b) $D_0 = 44g$

Take moments about C

$65\cancel{g} \times 2{\cdot}6 + 17\cancel{g} \times 3 = 44\cancel{g} \times s$

$220 = 44s \Rightarrow \underline{\underline{s = 5}}$

~~R~~ D is 5m from C.

So $5+1$ metres $= 6$ m from A

Riverbank is $1 + 0{\cdot}6 + 4 = 5{\cdot}6$ m from A

$\Rightarrow$ D is $0{\cdot}4$m from nearest riverbank

(c) I have assumed that centre of mass of the plank is directly in the middle of the plank.

(5) (a) 90 km h^{-1}

$(\times 1000) = 90000 \text{ m h}^{-1}$

$(\div 3600) = 25 \text{ ms}^{-1}$

(b) At max speed $\Rightarrow$ max power output and $\underline{\underline{a = 0}}$.

F_D = total resistive forces

$= 400 \times 5 + 3000 = 5000$ N

$P = F_D v = 5000 \times 25 = 125000$ W

$= 125$ kW

6 (a) $F = -2mv^{\frac{5}{4}}$

$a = \frac{F}{m} = -2v^{\frac{5}{4}}$

$\Rightarrow \frac{dv}{dt} = -2v^{\frac{5}{4}}$

(b) $\frac{dv}{dt} = -2v^{\frac{5}{4}}$

$\int \frac{1}{v^{\frac{5}{4}}}\,dv = -\int 2\,dt \quad \Rightarrow \quad \int v^{-\frac{5}{4}}\,dv = -\int 2\,dt$

So $\frac{v^{-\frac{1}{4}}}{-\frac{1}{4}} = -2t + c$

$-4v^{-\frac{1}{4}} = -2t + c$

$4v^{-\frac{1}{4}} = 2t + d$ $(\times -1)$

$t=0 \quad v=16$

$4(16)^{-\frac{1}{4}} = 0 + d$

$\frac{4}{2} = d \quad \Rightarrow d = 2$

$4v^{-\frac{1}{4}} = 2t + 2$

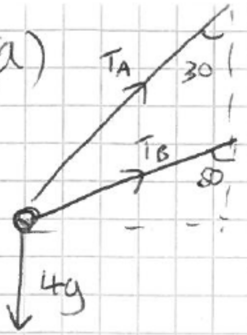
$v^{-\frac{1}{4}} = \frac{1}{2}t + \frac{1}{2}$

$\frac{1}{v^{\frac{1}{4}}} = \frac{1}{2}t + \frac{1}{2} \quad \Rightarrow \quad v^{\frac{1}{4}} = \frac{1}{\frac{1}{2}t + \frac{1}{2}}$ $\frac{\times 2}{\times 2}$

$v^{\frac{1}{4}} = \frac{2}{t+1}$

$\Rightarrow \quad v = \left(\frac{2}{t+1}\right)^4$

7 (a)



$\leftarrow T_A\cos 30$

$\leftarrow T_B\cos 50 = 20\sin 50$

resolve verti[cally]

$4g = T_A\cos 30 + 2[0\cos 50]$

$\dfrac{4g - 20\cos 50}{\cos 30} = T_A$

$\Rightarrow T_A = 30.4197$

$= 30.4N$

(b) Net force inwards $= T_A\sin 30 + T_B\sin 50$

$= 30.4197 \times \sin 30 + 20\sin 50$

$= 30.5307\ N$

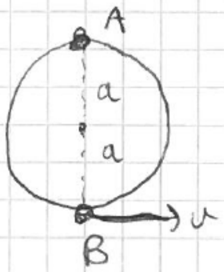
$F = \dfrac{mv^2}{r}$

$F = \dfrac{4\times 5^2}{r} \Rightarrow r = \dfrac{4\times 25}{F} = \dfrac{100}{30.5307}$

$r = 3.275387$

$r = \underline{\underline{3.28m}}$

8 (a) it needs to have a high enough initial E_k so that it can gain enough GPE to reach the top of the circle.



$\Delta h = 2a$

$mg\Delta h = 0.3\times g\times 2a = 0.6ag$

so $0.6ag$ is the required initial E_k

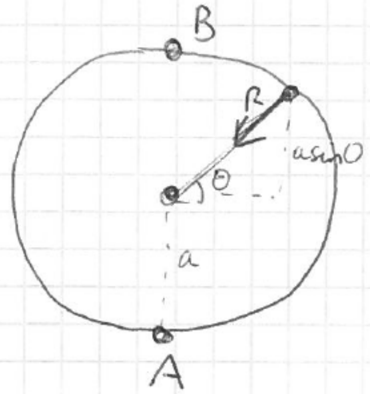
let u be min initial speed $\therefore$ $0.6ag = \frac{1}{2}mu^2$

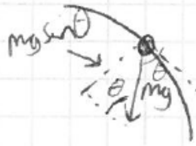
$1.2ag = 0.3u^2$

$\Rightarrow \sqrt{\dfrac{1.2ag}{0.3}} = u^2 \quad \Rightarrow u^2 = \sqrt{4ag} = \sqrt{4}\sqrt{ag} = 2\sqrt{ag}$

so $u > 2\sqrt{ag}$ ooc

b(i)





Net force (inwards radially) $= R + mg\sin\theta$

(1) $mg\sin\theta + R = \dfrac{mv^2}{r}$

E_k at B $= \frac{1}{2}mu^2 - \Delta GPE$

$\Delta h = a + a\sin\theta$ (between A and B)
$= a(1+\sin\theta)$

so $\Delta GPE = mg\Delta h = mga(1+\sin\theta)$

so $\frac{1}{2}\cancel{m}v^2 = \frac{1}{2}\cancel{m}u^2 - \cancel{m}ga(1+\sin\theta)$

$v^2 = u^2 - 2ag(1+\sin\theta)$

$v^2 = \frac{9}{2}ag - 2ag - 2ag\sin\theta$

$= \frac{5}{2}ag - 2ag\sin\theta$

$\left(v = \sqrt{\frac{5}{2}ag - 2ag\sin\theta}\right)$

(1) $mg\sin\theta + R = \dfrac{m}{\cancel{a}}\left(\frac{5}{2}\cancel{a}g - 2\cancel{a}g\sin\theta\right)$

$mg\sin\theta + R = \frac{5}{2}mg - 2mg\sin\theta$

$R = \frac{5}{2}mg - 3mg\sin\theta$

$\underline{m = 0.3}$

$= \frac{3}{4}g - \frac{9}{10}g\sin\theta$

$R = g\left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)$

(ii) $R=0$

$$0 = g\left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)$$

$$\frac{9}{10}\sin\theta = \frac{3}{4}$$

$$\sin\theta = \frac{0.75}{0.9} \Rightarrow \theta = \sin^{-1}\left(\frac{0.75}{0.9}\right)$$

$$= 56.4427$$

$$\theta = 56.4^\circ$$

(above the horizontal.)

(9) $L = 6\text{m}$ $\quad \lambda = 1800\text{N}$

(a) $EPE = \frac{\lambda x^2}{2L}$ $\quad x = 10 - 6 = 4\text{m}$

$$= \frac{1800 \times 4^2}{2 \times 6} = 2400\text{J}$$

(b) Initial $E_k = \frac{1}{2}mu^2 = \frac{1}{2} \times 200 \times 8^2 = \underline{6400\text{J}}$

$$E_k \rightarrow EPE$$

$$6400 = \frac{\lambda x^2}{2L} = \frac{1800x^2}{2 \times 16}$$

$$6400 = \frac{1800}{12}x^2$$

$$\frac{128}{3} = x^2$$

$$\Rightarrow x = 6.532\text{m}$$

distance from $O = 6 + x = 12.5\text{m}$ (3SF)

(c) Now $E_k \rightarrow GPE$ + work done against resistive forces.

$$6400 = \frac{1800x^2}{2\times 6} + Fs$$

where $F = 800N$ and s = distance moved $= l + x$
$= 6 + x$

so
$$6400 = 150x^2 + 800(6+x)$$
$$6400 = 150x^2 + 4800 + 800x$$
$$\Rightarrow \quad 0 = 150x^2 + 800x - 1600$$
$$(\div 50) \quad 0 = 3x^2 + 16x - 32$$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(3)(-32)}}{2\times 3} = \frac{-16 \pm \sqrt{640}}{6}$$

$x = 1.5497$ or $x = -6.88$

$\downarrow$

distance from O $= 6 + x$

$= \underline{\underline{7.55\,m}}$

$\underline{\underline{(3SF)}}$